Simultaneous Kinematics

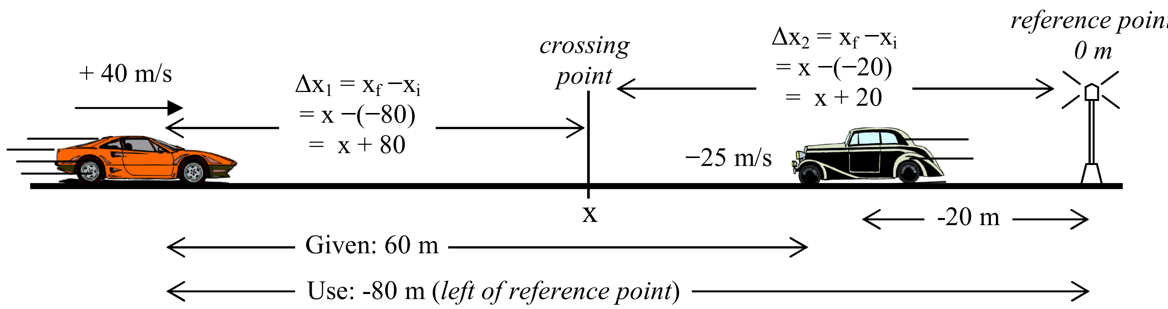
When multiple objects are moving it is possible to use multiple kinematic equations to solve for multiple unknowns.

Common Reference Point

To solve simultaneous equations all variables must relate to each other. This can only happen if they are set up with reference to a common point, which could be a fixed object or the far left or right point of the situation.

Ex 1. A sports car is traveling 40 m/s to the right. A luxury car is traveling 25 m/s to the left. If the sports car starts 60 m to the left of the luxury car and the luxury car starts 20 m to the left of the light post, where do they cross?

Step 1—Common reference point: the light post. Give all positions relative to the light post. Then "60 m left of the luxury car" becomes "−80 m" (negative because it is to the left of the light post).



$$v_{sc} = \frac{\Delta x}{t}$$
$$t = \frac{\Delta x}{v_{sc}}$$
$$t = \frac{x+80}{40}$$

Why did we solve for "t"? Because we are looking for x. If we did solve for t, we can plug t back into either equation and then solve for x.

$$v_{lc} = \frac{\Delta x}{t}$$
$$t = \frac{\Delta x}{v_{lc}}$$
$$t = \frac{x+20}{-25}$$

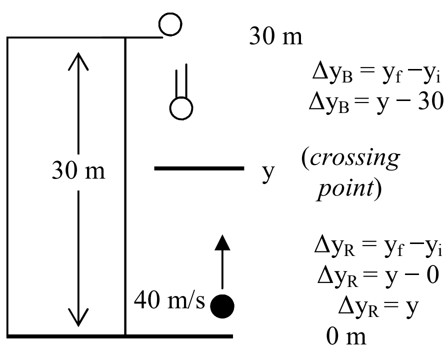
$$t_{sc} = t_c$$
$$\frac{x+80}{40} = \frac{x+20}{-25}$$
$$-25(x+80) = 40(x+20)$$
$$-25x - 2000 = 40x + 800$$

$$-2000 - 800 = 40x + 25x$$
$$-2800 = 65x$$
$$x = \frac{-2800}{65} = \boxed{-43.1\text{ m}}$$

Use Normal Directions

Simultaneous equations are too complicated to add difficulties. Keep positives up and to the right, otherwise you may make mistakes.

Ex 2. A ball is dropped from 30m. At the same time a rock is thrown from the ground going 40 m/s. Where do they cross?



Ball:

$$\Delta y = v_i t + \frac{1}{2}at^2$$
$$y - 30 = 0t + \frac{1}{2}(-9.8)t^2 \quad \text{Eq 1:}$$

Rock:

$$\Delta y = v_i t + \frac{1}{2}at^2$$
$$y = 40t + \frac{1}{2}(-9.8)t^2 \quad \text{Eq 2:}$$

Adding Equations

Anything done to one side of an equation must be done to the other side to keep the equation equal. Instead of substituting, you can add equations together. Since it is an equation, you are still adding the same amount to both sides. This can be much easier than substitution.

Notice that the right most term is the same in each equation. If one of them was negative, adding the two equations would remove the t^2 term, making solving much easier.

$$-(y-30) = -\left(0t + \frac{1}{2}(-9.8)t^2\right)$$
$$-y + 30 = -\frac{1}{2}(-9.8)t^2$$

Add the equations:

$$y = 40t + \frac{1}{2}(-9.8)t^2$$
$$+$$
$$-y + 30 = -\frac{1}{2}(-9.8)t^2$$
$$=$$
$$30 = 40t$$
$$t = \frac{30}{40} = .75\text{ sec}$$

Even though we are solving for y, it is easier to solve for t mathematically.

Now solve for y using either equation.

$$y = 40(.75) + \frac{1}{2}(-9.8)(.75)^2$$
$$y = 30 - 4.13 = 27.2\text{m}$$