Thermal Equilibrium Problems

A hot object is put in contact with a cold object, find the final temperature.

Thermal equilibrium is defined as when objects are at the same temperature. When a cold and hot object are put together they will eventually come to thermal equilibrium. This final temperature will be somewhere between the two starting temperatures. This should be obvious: the cold object will heat up; the hot object will cool down.

It should also be obvious that the amount of heat lost by the hot object will be given to the cold object.

Q_{hot} transferred to Q_{cold} (Law of Conservation of Energy)

It may seem that $Q_{lost} = Q_{gained}$, but this is not completely correct.

Q_{lost} is negative (heat goes out); Q_{gained} is positive (heat comes in). Since + ¹ – we need to either make the positive side negative, or the negative side positive. You know you can do this by multiplying one side by a negative. Fortunately you don't even need to remember which side has the negative, because a negative on either side makes the signs of the sides equal.

2 choices: –(–) = + OR –(+) = –

So, our equation becomes:

$-Q_{lost} = Q_{gained}$ OR $Q_{lost} = -Q_{gained}$

(Or use the Law of Conservation of Energy: $\Delta E = 0$(closed system) $= Q_{lost} + Q_{gained}$, then $Q_{lost} = -Q_{gained}$.)

Either one is correct. So let's make it easier:

$Q_{object1} = -Q_{object2}$ which will work for any two-object system.

You should already know that $Q = mc_p\Delta T$, that $\Delta T = T_f - T_i$, and that the specific heat (c_p) of water is 1 cal/g^{o}C.

Example Problem: A 12 gram piece of aluminum (c_p = .215 cal/g^{o}C) is at 70^{o}C. It is placed in a beaker that contains 35 grams of 15^{o}C water. At what temperature will they come to thermal equilibrium?

(Could also say: "Find the final temperature of the mixture.", etc.)

$Q_{object1} = -Q_{object2}$	*(negative on either side)*
$Q_{aluminum} = -Q_{water}$	*(makes the equation specific to this problem)*
$m_A c_{pA} \Delta T_A = -m_w c_{pw} \Delta T_w$	*(the subscripts help us keep track of our variables it may seem like a hassle, but it reduces errors significantly)*
$m_A c_{pA}(T_f - T_{iA}) = -m_w c_{pw}(T_f - T_{iw})$	*(putting in the equation for ΔT)*
$(12g)(.215)(T_f - 70^oC) = -35(1)(T_f - 15^oC)$	*(putting in our numbers from the problem; notice c_p water is 1 cal/g^{o}C, even though it was not given)*
$(2.58)(T_f - 70) = -35(T_f - 15)$	*(dropped units to make the algebra easier)*
$2.58\,T_f - 180.6 = -35T_f + 525$	*(distributive property of multiplication; make sure the 525 is positive now [– times – = +])*
$37.58T_f = 705.6$	*(combining like terms)*
$T_f = 18.8^oC$	*(put units back at the end)*

Does this answer make sense? Yes: because 18.8^{o}C is between the two objects' initial temperatures of 70^{o}C and 15^{o}C. The cold object heated up and the hot object cooled down. If you did not have negative or made a critical algebra error, the final temperature might be outside this range, which makes no sense.